Quantifying the Impact of Uncertain Material Parameters on Pavement Response using an Inverse Modeling Technique

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Abstract – Accurate modeling of pavement response plays a critical role in the effective design, analysis, and maintenance of road infrastructure. However, the presence of uncertainty in material parameters can significantly compromise the reliability and accuracy of such models. This study focuses on investigating the impact of uncertain material parameters on pavement response by employing an inverse modeling technique. The objective of this research is to utilize an inverse modeling approach to assess the influence of uncertain material parameters on Uzan's model, a commonly used model for pavement response. The study considers measured stress and strain values obtained from tyre and Falling Weight Deflectometer (FWD) load conditions applied to granular materials. The inverse model is formulated as a nonlinear least squares minimization problem, in conjunction with a finite element model that analyzes the deformation of flexible pavements. Through the application of the inverse modeling technique, this study aims to determine the extent to which uncertain material parameters affect the accuracy of pavement response predictions. By comparing the predicted pavement behavior derived from the inverse model with actual measured data, the influence of uncertain parameters can be quantified. The outcomes of this research contribute to advancing the understanding of the complex interplay between material parameter uncertainties and pavement response.

Keywords: Finite element modeling, Inverse problem, Parameter estimation, Pavement response.

I Introduction

The accuracy of modeling the response of pavement is greatly influenced by the material model employed, particularly the material constants associated with it. Any inaccuracies in the chosen model can result in discrepancies between the calculated behavior and the actual observed behavior of the pavement.

Numerical modeling based on displacement is a valuable tool in pavement design, allowing for the simulation of the mechanical response of materials under axle loads. Within the realm of flexible pavements, various deformation models exist in the literature for studying and modeling permanent deformation. Many of these models rely on material models that consider stress levels and stress histories. One commonly used model is Uzan's equation (Equation 1) [4]. Uzan's model effectively incorporates both the confining and wheel load deviator stresses, enabling accurate representation of resilient modulus changes within unbound aggregate base layers. Material constants utilized in the model are typically derived from laboratory repeat load triaxial (RLT) test measurements.

Werkeister et al.[8] compared Dresden and Uzan nonlinear material models using data from pavement tests at Transit New Zealand's CAPTIF facility. Uzan model fit well for vertical and transverse strains at one station but struggled with longitudinal strains. Both models had discrepancies in deflection predictions. The study highlighted limitations in accurately predicting pavement response using these models, particularly for longitudinal strains and deflections at various locations within the pavement structure. Further research and refinement are needed to enhance accuracy and reliability in capturing the complex behavior of pavements under different loading conditions.

Steven et al.[5] calibrated their FEM using the Uzan model with FWD test data. Initially, the computed deflection was 31% higher than measured values, but adjusting the k_1 value improved agreement. However, this led to an 81% difference in vertical strain values. Through iterative adjustments to k_1, they fine-tuned the model, achieving accurate representation of pavement response. This calibration process emphasizes refining material parameters to enhance numerical model accuracy and reliable pavement behavior predictions.

Meshkani et al.[4] developed an algorithm to optimize deflection basins from FWD measurements and calculate pavement profile using a constitutive model. They conducted a sensitivity analysis on nonlinear parameters of the base and subgrade, examining their impact on pavement response. Deflections under load and initial measurement points were highly sensitive to parameter variations, while the remaining measurements were less affected. The study emphasized the significance of considering nonlinear parameters to accurately predict pavement response. By identifying critical areas influenced by these parameters, the research provided insights for optimizing pavement design and maintenance strategies. Understanding pavement response sensitivity is crucial for ensuring long-term performance and durability.

In 2008, Tutumluer [6] provided an overview of the advancements in characterizing the anisotropic properties of unbound aggregate layers. The objective was to contribute to the development of future highways by utilizing extensive knowledge accumulated over the previous 15 years regarding stress distribution and the anisotropic behavior of aggregates. The research article emphasized that under wheel load, the anisotropic ratio was relatively low. However, as the distance from the centerline increased, the ratio exhibited a significant increase, eventually reaching a value of 1, which corresponds to an isotropic scenario.

In 2009, Kim et al. [3] showcased the latest research conducted at the University of Illinois, which revolved around the application of specific-purpose axisymmetric and general-purpose three-dimensional (3D) finite element (FE) programs for analyzing flexible pavements. The study aimed to predict pavement responses by incorporating a linear elastic asphalt layer and accounting for nonlinearity in the base and subgrade layers. Notably, the contour plot of the vertical resilient modulus revealed minimal and negligible variations within the subgrade layer compared to the base course layer. This finding shed light on the relatively insignificant influence of modulus fluctuations within the subgrade layer in the context of the research's pavement analyses.

Tarefder et al.[6] conducted a study to explore the effects of anisotropy on pavement deformation, specifically focusing on horizontal strain at the bottom of the asphalt layer and vertical strains within all layers (asphalt, base, sub-base, and subgrade). The research aimed to assess the impact of anisotropy on these strains. The findings revealed that the vertical strain in the asphalt layer exhibited low sensitivity to the anisotropy of unbound layers. On the other hand, the vertical strains in the base, sub-base, and subgrade layers were found to be significantly influenced by anisotropy, with the exception of the subgrade where anisotropy had only a slight effect on the vertical strain.

In our next phase, we undertook a thorough evaluation of the performance of a recently reconstructed pavement. This evaluation specifically focused on analyzing falling weight deflectometer (FWD) measurements obtained from two pavement sections: the pre-existing pavement and the newly reconstructed pavement situated in Jaffna, Sri Lanka. Our primary objective was to assess the effectiveness and quality of the reconstruction project by carefully comparing the FWD measurements from these two sections. Figure 1 depicts the variations in the deflection ratio throughout the pavement's length, serving as a measure to assess its strength [3]. Notably, specific segments of the recently reconstructed pavement continue to display deflection ratios below 0.6. According to Gallage et al. [3], a deflection ratio exceeding 0.6 signifies a pavement of satisfactory quality. Consequently, these observations imply that the recent reconstruction endeavors have not adequately bolstered the pavement's strength.

The design of granular pavements remains a complex task, primarily because of the inbuilt challenges in accurately determining the material parameters necessary for an accurate simulation of pavement response models. The difficulty in identifying suitable values for these material parameters highlights the complicated nature of designing granular pavements that effectively meet desired performance standards.

The primary aim of this research paper is to investigate the influence of material constants on pavement deformation. To accomplish this objective, a finite element pavement response model is utilized, incorporating Uzan's non-linear stress-dependent material model.

In order to enhance the precision of the model, an iterative least squares minimization procedure is implemented to fine-tune the material equation constants, with the objective of achieving a more optimal alignment with the measured data. The efficacy of this inverse modeling approach is evaluated by integrating the optimized material equation constants into the finite element model. This facilitates the prediction of strains in the pavement under different tire loads, which are then compared to the corresponding measured strains.

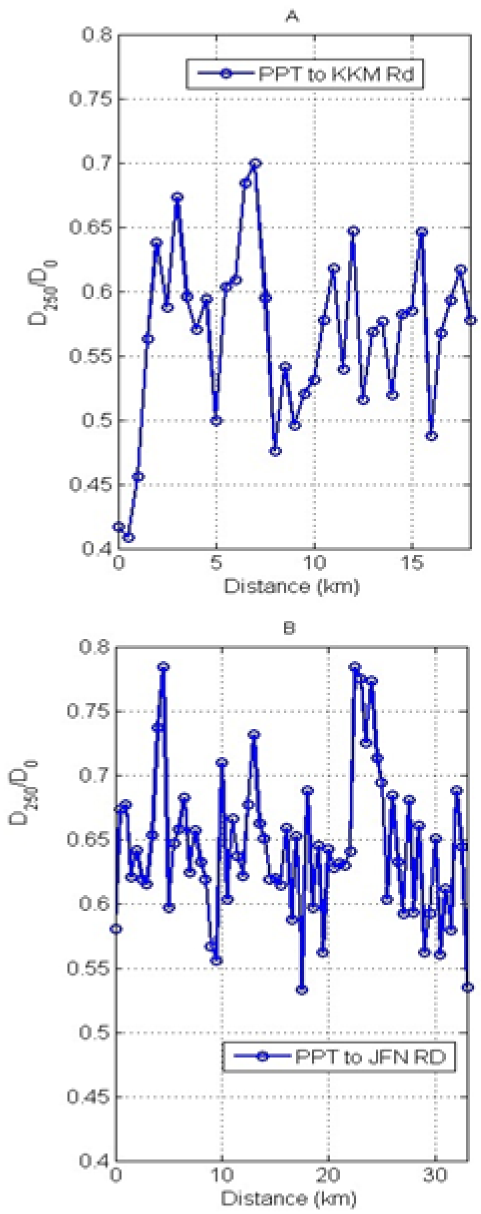

Figure 1: (A) Old Pavement, (B) Newly constructed pavements

II Forward Model

Prior to delving into the numerical optimization of material parameter estimation, it is imperative to precisely formulate the forward problem. This problem entails determining the stress-strain characteristics by considering specific material parameters and loading conditions. To achieve this objective, a widely utilized general-purpose finite element (FE) software, ABAQUS (Standard version 6.7), has been utilized to construct a comprehensive three-dimensional forward finite element model.

To ease computational complexity, the geometry's symmetry is used, and only one-quarter of the model, as illustrated in Figure 2, is taken into account. Exploiting this symmetry significantly reduces computational effort, enhancing the efficiency and manageability of the analysis. This approach ensures the accurate representation of the system's behavior, enabling the acquisition of precise stress-strain profiles corresponding to the provided material parameters and loading conditions.

The present study utilizes a finite element (FE) model comprising three distinct layers: the asphalt surface, base course, and subgrade. The asphalt layer consists of stones and bitumen, with a height of 40 mm.

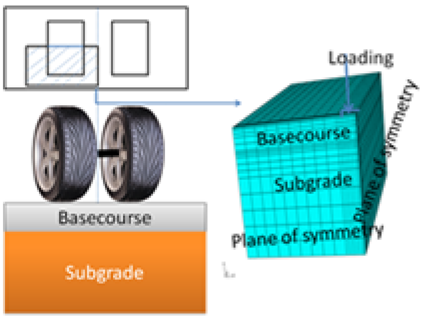

Figure 2: Considered model

The stones are treated as rigid bodies, while the bitumen is represented using hyperelastic and viscoelastic material models, which have been previously explained in our earlier publication [2]. Additionally, a linearly elastic material model is employed for the bitumen to simplify the analysis. Both the base course and subgrade are considered deformable and possess anisotropic material properties. The base course has a height of 300 mm, whereas the subgrade measures 1200 mm in height. The modulus of elasticity for these layers is calculated using equation 1 as outlined in reference [3].

$$E_1 = k_1 p_a \left(\frac{\theta}{p_a}\right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1\right)^{k_3} \tag{1}$$

where $\theta = \sigma_1 + \sigma_2 + \sigma_3$
$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$
$p_a = 100\,\text{kPa}$
$E_1 =$ Modulus of elasticity
k_1, k_2 and k_3 are constants.

In an isotropic model, the modulus remains consistent in all directions. However, an anisotropic model exhibits varying material properties, such as modulus and Poisson's ratio, in the horizontal and vertical directions. The constitutive relation for stress (σ) and strain (ε) in the presence of cross anisotropy can be mathematically represented as expressed in [2].

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} D_{1111} & D_{1122} & D_{1133} & 0 & 0 & 0 \\ D_{1122} & D_{2222} & D_{2233} & 0 & 0 & 0 \\ D_{1133} & D_{2233} & D_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{1212} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{1212} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{bmatrix}$$

where

$$\begin{aligned} D_{1111} &= E_1(1 - \nu_{13}\nu_{31})\lambda = D_{2222} \\ D_{3333} &= E_3(1 - \nu_{12}\nu_{12})\lambda \\ D_{1122} &= E_1(\nu_{12} + \nu_{31}\nu_{13})\lambda \\ D_{1133} &= E_1(\nu_{31} + \nu_{12}\nu_{31})\lambda = D_{2233} \\ D_{1212} &= \frac{E_1}{2(1 + \nu_{12})} = G_{12} \\ D_{1313} &= D_{2323} = G_{13} \\ \lambda &= \frac{1}{1 - \nu_{12}^2 - 2(\nu_{13}\nu_{31}) - 2(\nu_{12}\nu_{13}\nu_{31})} \\ n &= \frac{E_1}{E_3} = \frac{\nu_{13}}{\nu_{31}} \end{aligned}$$

$G_{13} =$ shear modulus in 1–3 plane, $E_1 =$ modulus in the plane of isotropy, $E_3 =$ modulus normal to the plane of isotropy, $\nu_{12} =$ Poisson's ratio for strain in direction 1 due to strain in direction 2, $\nu_{13} =$ Poisson's ratio for strain in direction 1 due to strain in direction 3, $\nu_{31} =$ Poisson's ratio for strain in direction 3 due to strain in direction 1.

The above mentioned material model is employed in ABAQUS through the utilization of a user-written subroutine called UMAT, specifically designed for basecourse and subgrade analysis. This subroutine enables the definition of field variables at each material point within an element and grants access to a range of variables, including stress and strains.

III Parameter Estimation

The initial stage involves extracting the material model constants from field measurements of strain obtained at various points within an actual pavement. Mathematically, this challenge is recognized as an inverse problem, akin to an optimization problem. The objective function to be minimized in this context is the disparity between the measured strain values and the estimated values derived from finite element (FE) simulations at a specific location.

To determine the global minimum within a given search space, an objective function is essential to mathematically define the disparities between the measured and simulated outcomes. This objective function is then minimized while considering any applicable constraints. The problem can be formulated as follows:

$$\text{Minimize} \quad f(\boldsymbol{p}) = \sqrt{\sum_{i=1}^{n} \left[\varepsilon_i - \hat{\varepsilon}_i(\boldsymbol{p})\right]^2}$$

Subject to:

$$g_j(\boldsymbol{p}) \geq 0 \qquad j = 1, 2, \ldots, n_1 \tag{2}$$
$$h_k(\boldsymbol{p}) \geq 0 \qquad k = 1, 2, \ldots, n_2 \tag{3}$$

where $f(\boldsymbol{p})$ is the objective function, $g_j(\boldsymbol{p})$, $h_k(\boldsymbol{p})$ are constraint functions, $\boldsymbol{p}$ is a vector of constants and E is the log of measured strain values and E is the log of modeled strain values, n_1 and n_2 are number of constraints. For the optimal fit, $\boldsymbol{p}$ must be varied to minimize $f(\boldsymbol{p})$.

As a nonlinear problem, there is a possibility of convergence to a local minimum if the initial values are not sufficiently accurate. Initial simulation experiments using simulated field data revealed that the procedure yielded different solutions for various initial values. To address the issue of local minima, an approach was employed where the error term was calculated for a range of constants at a wider interval. Subsequently, the minimum value among the obtained minima was selected. In the subsequent step, the constant values associated with that minimum were utilized as the new starting values.

To verify the precision of the nonlinear inverse model employed in the ABAQUS finite element program, an initial validation process was conducted to assess its capability in predicting pavement responses. Using relevant literature as a reference, arbitrary yet realistic values for k_1, k_2, k_3, and n were chosen. These values were then employed in the forward model to predict the strain data within the modeled pavement. Subsequently, these strain values were treated as "field" data within the inverse model to calculate the original constant values.

IV Results and Discussions

The data utilized in this study were obtained from real accelerated traffic loads conducted at CAPTIF (The Canterbury Accelerated Pavement Testing Indoor Facility, New Zealand). Specifically, the field measurements of pavement responses, including strain and deflection, from the CAPTIF test measurements were employed. To examine the impact of material parameter values, five sets of experimental data from four distinct pavements were considered to showcase the developed method. The composition of the pavement sections under consideration is depicted in Table 1.

Table 1: Composition of pavements sections of considerations

Pavement	Surfacing	Granular Base	Subgrade
1	30	277	1193
2	34	144	1322
3	40	150	1310
4	37	300	1163

To initiate the process, the vertical elastic strains, obtained from measurements conducted under a 40 kN dual wheel load with a pressure of 650 MPa, were utilized as inputs in the inverse model to compute the material parameters for pavement 1. These values are presented in Table 2. The entire document should be in Times New Roman. Other font types may be used if needed for special purposes.

Table 2: Optimal values of constants

Asphalt layer modulus	5000 MPa
Subgrade modulus	40 MPa
k_1	4500
k_2	2.10
k_3	-.30
n	.25

Subsequently, these determined parameter values were fed into the forward model to estimate the vertical strain at various depths under a 40 kN dual tire load with a pressure of 750 kPa. The measured vertical strain data, which were employed for this analysis, are presented in Table 3.

Table 3: Measured strain against depth for 40 kN dual tyre with 650 kPa, 750 kPa of pavement 1

Depth (m)	Microstrain 650 MPa	Microstrain 750 MPa
.1125	880	880
.1875	600	720
.2625	720	780
.3375	2160	2240
.4125	2400	2500
.4875	2000	2100
.5625	920	1000

Figure 3 illustrates the comparison between the calculated strain values and the corresponding measured values for a 40 kN dual tire load with a pressure of 750 MPa.

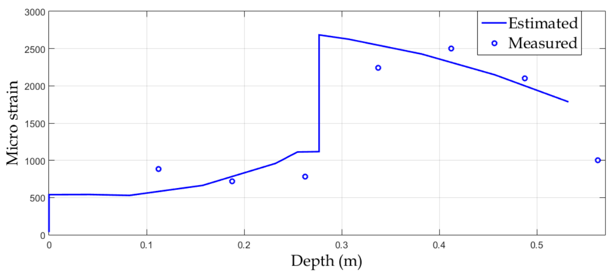

Figure 3: Calculated and measured strain values for 750 kPa tyre

Table 4 displays the pavement deflection measurements for pavement 1, which were obtained using the Falling Weight Deflectometer (FWD). These measurements were conducted after subjecting the pavement to one million cycles of a 40 kN axle load. The average surface deflection was recorded at various horizontal points along the surface under a 42 kN FWD load, equivalent to 595 kPa.

Table 4: FWD data – Pavement 1

Surface deflection under 42 kN, 595 kPa						
Distance from center (mm)	0	200	450	600	900	1800
Vertical displacement (mm)	1.062	0.738	0.282	0.165	0.079	0.038

Figure 4 depicts the comparison of FWD generated deflection and the calculated deflection for the material parameter obtained from this experiment.

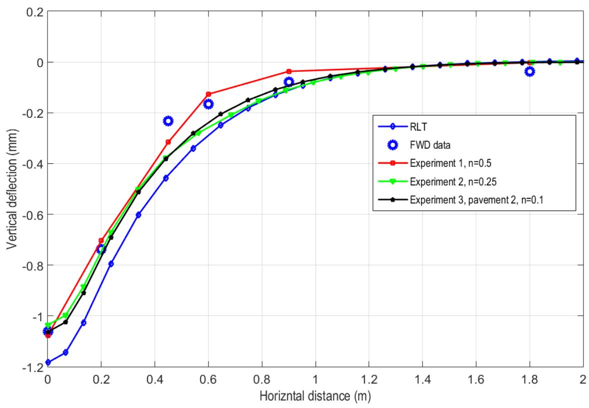

Figure 4: Comparison with FWD measurement

Table 5 depicts the parameter values calculated from FWD data alongside with parameter values based on repeated triaxial experiments.

Table 5: Optimal values of constants compared with the default RLT values

	FWD data	RLT data
Asphalt layer modulus	3000 MPa	3000 MPa
Subgrade modulus	40 MPa	40 MPa
k_1	5500	1800
k_2	.70	.75
k_3	-1.50	-.35
n	.40	1.00

In the second experiment, we utilized the vertical elastic strains recorded under a 50 kN dual wheel with a pressure of 800 MPa for pavement 2, 3, and 4 as input for the inverse model to determine the material parameters. The strain data utilized for this analysis are presented in Table 6.

Table 6: Measured strain against depth for 50 kN dual tyre with 800 kPa of pavements 2,3 and 4

Depth	Microstrain in Pavements		
(mm)	2	3	4
87.5	2407	3333	603
162.5	2500	3333	603
237.5	3518	1228	603
312.5	1574	526	474
387.5	925	526	1939
462.5	462	526	1250

Table 7 displays the outcomes of the estimations for the material parameters.

Table 7: Estimated material parameter values of pavements 2,3 and 4

Parameter	Pavements		
	2	3	4
n	0.1	0.15	0.4
k_1	6600	5300	5300
k_2	1.5	1.5	1.5
k_3	-.4	-.4	-.4

The series of experiments conducted above demonstrates the dependence of estimation values on the type of data used, such as FWD deflection and strain under the tire. This dependence may be attributed to the anisotropic ratio. From the FWD data, we obtained an optimal anisotropic value of 0.5. Similarly, based on strain measurements under the wheel, the optimal anisotropic ratio ranges between 0.1 and 0.25. Notably, this value increases with the thickness of the base course material.

In our model, the anisotropic ratio is not spatially dependent. However, in reality, it exhibits a lower value under the wheel load and gradually increases away from the centerline until reaching a value of 1. Additionally, it increases vertically downward from the load, albeit at a slower rate compared to the radial direction. Figure 4 visually represents the variation of the anisotropic value in all three directions

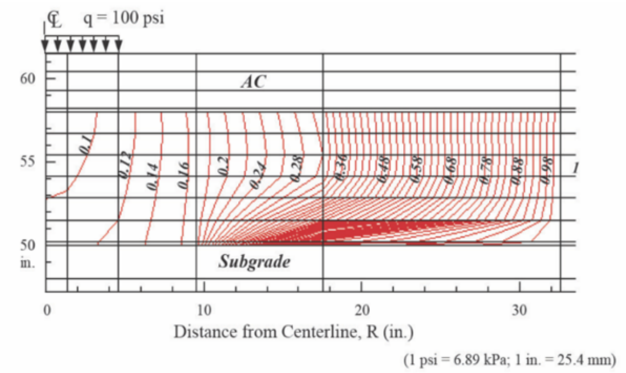

Figure 5: Horizontal stiffness ratio distribution throughout the base[6]

Figure 5 illustrates the results of a sensitivity analysis, demonstrating the impact of the anisotropic ratio (n) on the maximum deflection ($d0$), as well as the maximum strain in both the basecourse and subgrade.

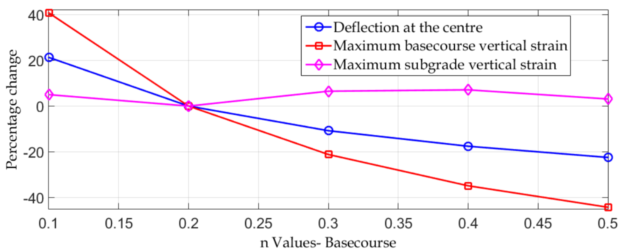

Figure 6: Percentage of change of maximum deflection, basecourse strain and subgrade strain against percentage of change of basecourse n.

V SUMMARY

The purpose of this paper is to present an inverse model that investigates the impact of values material constants on pavement design calculations. The approach is based on non-linear least squares estimation, utilizing simulated strain values in conjunction with the Finite Element technique.

In the final section, an illustrative example is provided to demonstrate how the model effectively determines the constants within the model. The results obtained from this example indicate that the aforementioned inverse model is capable of estimating the constants with a reasonable degree of accuracy.

Through the modeling process, it has been shown that the level of anisotropy significantly influences the estimations. Both Stevens in his PhD work[5] and Tutumluer[6] have found that the value n is approximately 0.2 for granular basecourses. This analysis has further confirmed this finding when considering strains directly under the wheel. However, when examining strains and deflection away from the wheel, the value of n tends to approach 1, indicating isotropy.

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