Adaptive Control of a Missile Interceptor Control System for Optimized Performance

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Introduction

The guided missile enables a military force enhanced capability or advantage over the enemy and even establish air superiority. This is due to its ability to deliver munitions precisely and rapidly to selected targets even at long ranges. Intercepting threats such as anti-ship cruise missiles or enemy aircraft require the ability to sense real time information on target location and quick reaction. Homing missiles, which operate on guidance decisions that are made via onboard sensors and computers, are used to intercept such unpredictable targets. The interception accuracy improves as the weapon closes in on the target since quality of information improves too. Even though modern Tactical Ballistic Missiles (TBM) has no capability of performing maneuvers they achieve a certain degree of inherent maneuvering potential due to high reentry speeds. This limited freedom is used to perform random hard maneuvers to avoid interception as the weapon closes in on the desired target. Anti ballistic missile defence systems such as PAC-3 and Arrow have displayed ability to efficiently intercept non maneuvering targets. Future research is motivated on developing a defence strategy that will guarantee that the miss distance resulting from an optimal evasive maneuver to be sufficiently small.

While the traditional method is to design separate subsystems for missile guidance and control systems and then integrate them, it does not fully exploit the synergetic relationships of interacting subsystems; thereby constraining overall performance. Integrated guidance and control on the other hand renders performance optimization. Such systems are usually time varying with unmatched uncertainties, mandating the controller to ensure the accuracy of target interception and the stability of the missile dynamics simultaneously. Min-Xhe et all (2010) developed an integrated guidance and control for homing missiles against ground targets where the proposed feedback controller ensures accuracy of target interception and stability of missile dynamics.

Problem formulation

This research focuses on optimizing the trajectory of a missile interceptor against a TBM and designing an integrated controller with no separation between the guidance and autopilot. Since the target in concern is a TBM, this work provides an extension of previously designed systems to moving airborne targets. The present investigation scenario is based on the following set of assumptions:

1) A flat non rotating earth has been assumed to neglect Coriolis and gravitation effects.
2) Both missile and threat are represented by point-mass models with linear control dynamics
3) Effects of climatic and atmospheric influences on the trajectory have been ignored
4) The scenario has been simulated in a two dimensional environment

Methodology

Part I-Optimizing the Trajectory

Trajectory optimization involves maximizing or minimizing measured performance within prescribed system boundaries. This can be numerically solved using Non Linear Programming (NLP) techniques.

Figure 1 shows a schematic view of the planar geometry of an oncoming threat and a weapon on pursuit. Remote sensors are deployed in relatively close proximity to sense the launch of the said missile. The horizontal and vertical components of position are denoted by *E* and *N* respectively and velocity is indicated by *V*. Subscripts *T* and *i* are used to denote the characteristics of the threat and interceptor. *R* is the distance between the threat and interceptor at any given point in time while *m* is the miss distance. χ denotes the interceptor angle to the vertical and χ_{LOS} denotes the Line of Sight Angle. The angle of the threat at burn out to the vertical is θ_{bo}.

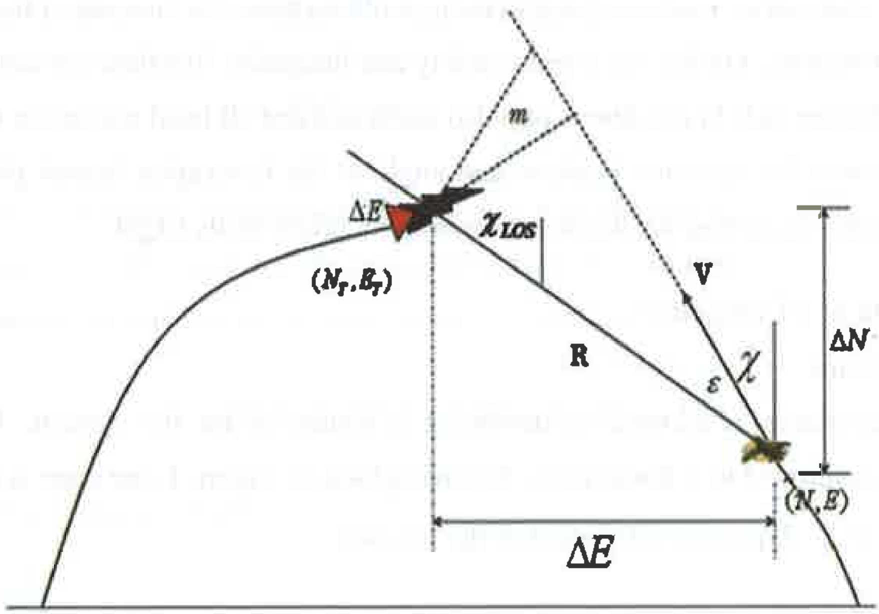


Figure 1. Planar Geometry of the TMB and interceptor in flight

In optimizing the trajectory, the goal is to find the launch angle, a_0, and time till intersection t_{if} that will maximize the interception range, $R_i = V_i.\cos a_0.t_{if}$.

i.e. Maximize $V_{bo}.\cos O_{bo}.(t_d - t_r - t_{if}) + X_{bo}.V_{bo}.\cos O_{bo}.t_{bf} + X_{bo}$

Note:- Time of TBM flight, $T_{bf} = t_{if} + t_r + t_d$,where t_r is the reaction time and t_d is the time of detection. V_{bo} is the velocity of the TBM at burnout and $V_{bxo}$and V_{byo} are its horizontal and vertical componants.

Equality Constraints are obtained considering the horizontal and vertical displacements of the threat and the interceptor respectively,

$$V_{bxo}.T_{bf} = V_{ixo}.t_{if} + E_i \longrightarrow (1)$$

$$V_{byo}.T_{bf} - gT_{bf}^2 + h_{bo} = V_{iyo}.t_{if} - gt_{if}^2 + 0 \longrightarrow (2)$$

Inequality Constraints are obtained considering the interception altitude and launch angle

$$h_{i_{min}} < H_i < maximum\ altitude\ of\ the\ interceptor \longrightarrow (3)$$

$$0 \leq a \leq 180^\circ \longrightarrow (4)$$

The problem is reduced to a convex programming problem since the Hessian of the cost function is positive semi definite. Further the linear equality and inequality functions are convex. Thus the Karush-Kuhn-Tucker (KKT) conditions are also sufficient and all local minimum are also global minimums. Thereby the optimum position and angle of the interceptor launch platform can be determined in order to destroy the threat with minimum effect on its target.

Part II – Design of a Controller

Dynamic Inversion

Feedback Linearization is adopted in designing a controller for the system. The system is algebraically transformed to a linear form. Referring back to Figure 1, the state is identified as χ and the control is $\dot{\chi}$. Equations of motion of the weapon:

$$\begin{bmatrix} \dot{N} \\ \Delta N \\ \dot{E} \\ \Delta E \end{bmatrix} = V \begin{bmatrix} \cos\chi \\ 0 \\ \sin\chi \\ 0 \end{bmatrix} + R \begin{bmatrix} 0 \\ \cos\chi_{LOS} \\ 0 \\ \sin\chi_{LOS} \end{bmatrix}$$

Change in distance between the threat and weapon:

$$\frac{dR^2}{dt} = 2R.R^2 = 2(N_T - N)(\dot{N}_T - \dot{N}) + 2(E_T - E)(\dot{E}_T - \dot{E})$$

$$\dot{R} = \cos\chi_{LOS}\left(\dot{N}_T - V\cos\chi\right) + \sin\chi_{LOS}\left(\dot{E}_T - V\sin\chi\right) \longrightarrow (5)$$

Change in the LOS,

$$\dot{\chi}_{LOS} = \frac{(E_T - E)\cos\chi_{LOS} + (N_T - N)\sin\chi_{LOS}}{R} \longrightarrow (6)$$

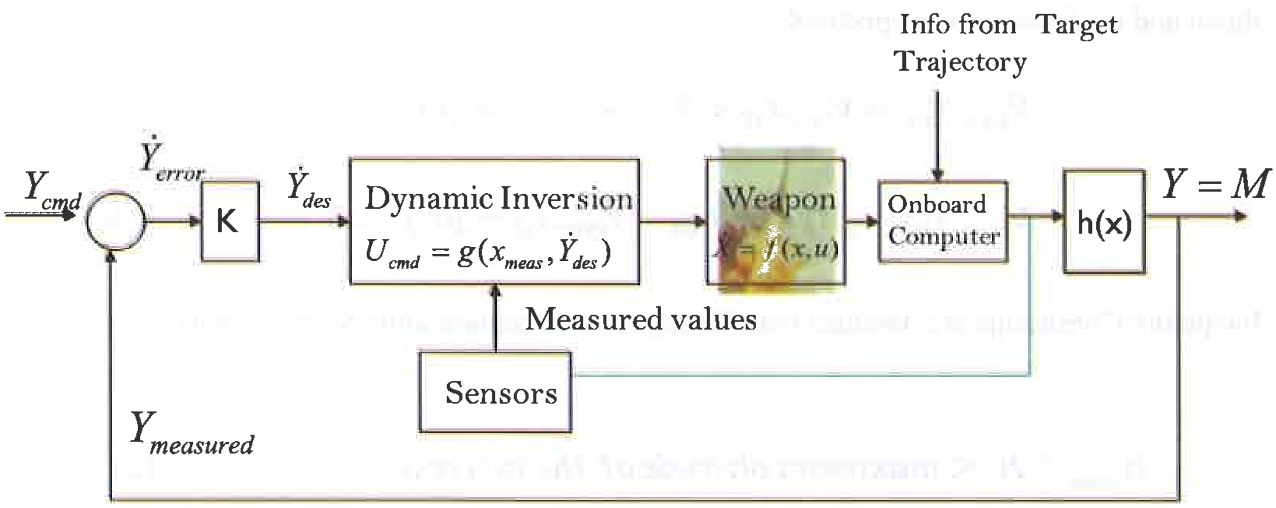


Figure 2. Basic System Block with Dynamic Inversion

Considering the system $\dot{X} = f(X,U) \longrightarrow (7)$

$$Y = h(X) \longrightarrow (8)$$

The objective is to make the output *y(t)* track a desirable trajectory $y_d(t)$ while keeping the whole system bounded, where $y_d(t)$ and its tune derivatives are assumed to be known and bounded. The miss distance is selected as a control variable. Considering Eqn.(8),

$$M = R \sin \varepsilon = R \sin(\chi_{LOS} - \chi) = Y$$

Examining Equation (5) and (6),

$$\dot{Y} = \frac{\partial Y}{\partial X}\dot{X} = \dot{R}\sin(\chi_{LOS} - \chi) + R\cos(\chi_{LOS} - \chi)(\dot{\chi}_{LOS} - \dot{\chi}) \longrightarrow (9)$$

This represents an explicit relationship between Y and U. If the control input is chosen in the form of $U_{cmd} = g\left(X_{meas}, \dot{Y}_{des}\right) \longrightarrow (10)$

and $\dot{Y} = f\left(X_{meas}, U_{cmd}\right) \longrightarrow (11)$

Let the tracking error, $e = Y(t) - Y_d(t)$ be the tracking error, and choosing a new input *v* as

$$v = \ddot{Y}_d - k_1\dot{e} - k_2 e = 0$$

With k_1 and k_2 being positive constants, the tracking error of the closed loop system is given by

$$\ddot{e} + k_2\dot{e} + k_1 e = 0$$

displays an exponentially stable error dynamics. Therefore if initially $e(0) = \dot{e}(0) = 0$, then $e(t) \equiv 0, \forall t \geq 0$, i.e., perfect tracking is achieved, otherwise e(t) converges to zero exponentially. It should be noted that the full state measurement is necessary in implementing the control law, because the computations of both the first derivative of Y and the input transformation required the value of χ.

Design of an Adaptive Controller

In order to adjust parameters online, an adaptive controller is preferred over an ordinary controller. The approach used in this research is to implement Model Reference Adaptive Control (MRAC), which can be depicted by the following schematic:

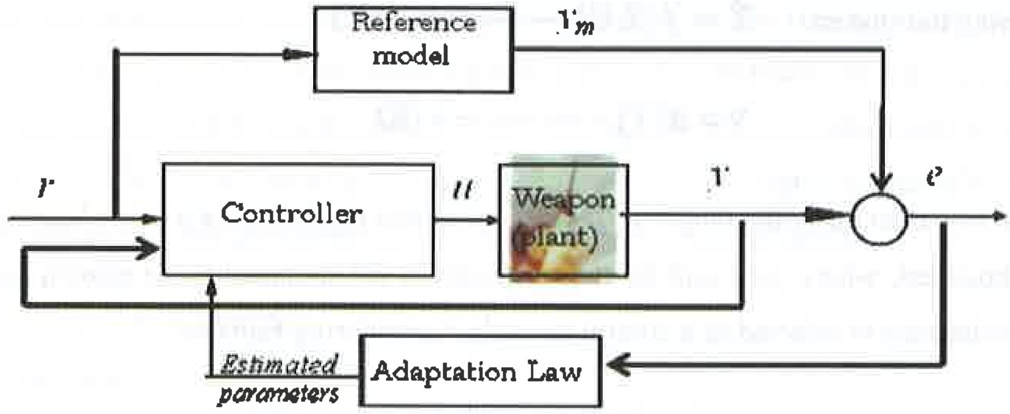


Figure 3. MARC system

The plant is the missile system in concern and the reference model is an ideal trajectory with the convergence of error to zero. i.e. the miss distance converging to zero, a perfect interception satisfying all constraints. The controller must have perfect tracking capacity in order to ensure tracking convergence. Even when plant parameters are not known, tracking convergence is achieved asymptotically. The adaptation mechanism is employed to adjust the control law parameters. It is important to synthesize an adaptation mechanism such that the control system stability is not compromised and the tracking error converges to zero even when parameters are varied.

The system the plant described in Eqn. (9) is approximated by the simple first order differential equation,

$$\dot{Y} = -a_p y + b_p u$$

Where y is the plant output and u is the input, and a_p and b_p are constant plant parameters. The desired system is approximated by

$$\dot{Y} = -a_m y_m + b_{mp} r(t)$$

where r(t) is a bounded external reference. Both and a_m and b_m are strictly positive to ensure stability of the reference model. Representing the transfer function of the reference model by M, it is possible to state that $y_m = Mr$ where $M = \frac{b_m}{p+a_m}$ with p being the Laplace variable.

The control law is approximated by $u = \hat{a}_r(t)r + \hat{a}_y(t)y$

where $\hat{a}_r$ and $\hat{a}_y$ are the variable feedback gains. Thus the closed loop dynamics can be written as

$$Y = -(a_p - \hat{a}_y b_p)y + \hat{a}_y b_p r(t)$$

This control law allows perfect matching of the model. When the online plant parameters are known (which can be easily obtained via onboard sensors of the missile), it leads to closed loop dynamics: $\dot{Y} = -a_m y + b_m r$ identical to the reference model and giving zero tracking error.

Let the tacking error be $e = y - y_m$. The difference between controller parameter provided by the adaptation law and the ideal parameters represent the error. i.e.,

$$\tilde{a}(t) = \begin{bmatrix} \tilde{a}_r \\ \tilde{a}_y \end{bmatrix} = \begin{bmatrix} \hat{a}_r - a_r^* \\ \hat{a}_y - a_y^* \end{bmatrix}$$

Thus the dynamics of the tracking error is obtained by:

$$\dot{e} = -a_m e + b_p(\tilde{a}_r r + \tilde{a}_y y)$$

Thereby the following adaptation law is derived:

$$\dot{\hat{a}}_r = -sgn(b_p)\gamma e r$$

$$\dot{\hat{a}}_y = -sgn(b_p)\gamma e y$$

with the adaptation gain, γ being a positive constant.

Results and Discussion

An example of the numerical analysis is shown in Table 1 for known inputs and guesses for the optimization problem. Usually V_{bo} of trajectories of are approximated in the range 2.9 Km/sec – 5.8 Km/sec. Thus for the TBM, V_{bo} is considered to be 4 Km/sec. The launch angle is set at 45°, and an altitude of 200 Km. The interceptor is at a constant speed equal to that of the threat. The t_d is 78 sec and t_r is 20 sec. The minimum interception attitude is set at 78 Km.

	V_{bo}=4, $V_{bo} = V_i$			V_{bo}=4, $\theta_{bo} = 45$			
	$\theta_{bo} = 45$	θ_{bo}=30	θ_{bo}=70	$V_i = 2V_{bo}$	$V_i = 0.75V_{bo}$	$V_i = 4.5$	$V_i = 3.5$
a - opt	1.2	1.09	1.22	0.93	1.5	1.44	1.09
t_r- opt	131.1	87	245	69.1	183	130	136
E_i	1632	1414	1049	1632	1632	2066	1250
Δt	355	289	437	577	577	650	505
ΔE	1004	1001	538	355	355	1227	807
E_t	2637	2415	1647	2637	2637	3293	2056
h_{imax}	204	204	204	408	153	230	178
$cost$	-787	-787	-787	-1575	-590	-887	-690

Table 1. Numerical Results of the Optimizes Trajectory

Analyzing the numerical outputs rendered by the simulation it is observed that E_i will increase when V_i is increased. Cost function is improved. E_i and t_{if} will decrease when V_{bo} is increased. Change in θ_{bo} will not affect the cost or h_i, but it will increase the time of flight of the missile. From intuition and from the results obtained it can be concluded that the range the interceptor will travel up to interception will increase when the average interceptor speed over ground is increased. Thereby it can be stated that the cost function is improved. The flight time of the interceptor to intercept the threat and the range travelled will decrease as the average ground speed of the threat is increased. Change in ϑ_{bo} will not affect the cost function. Figure 4 displays the simulation results for the implemented system in MATLAB (Simulink).

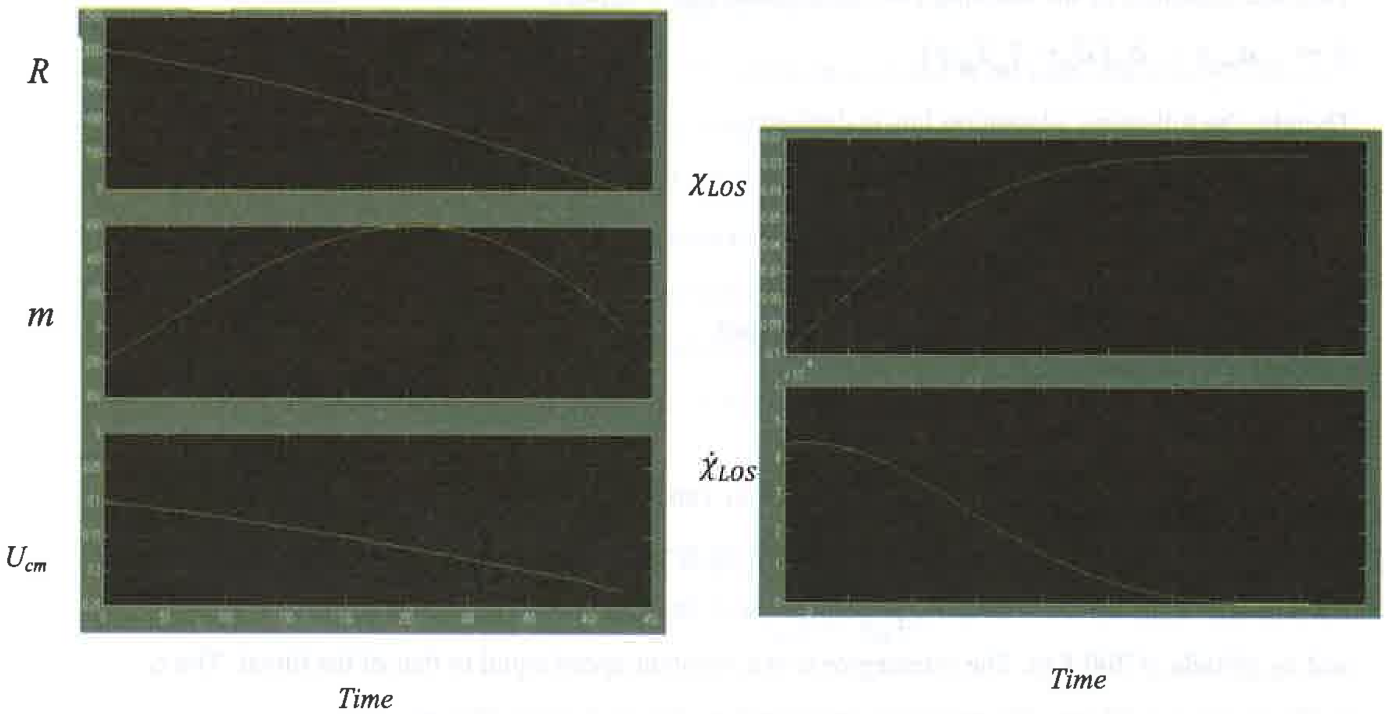
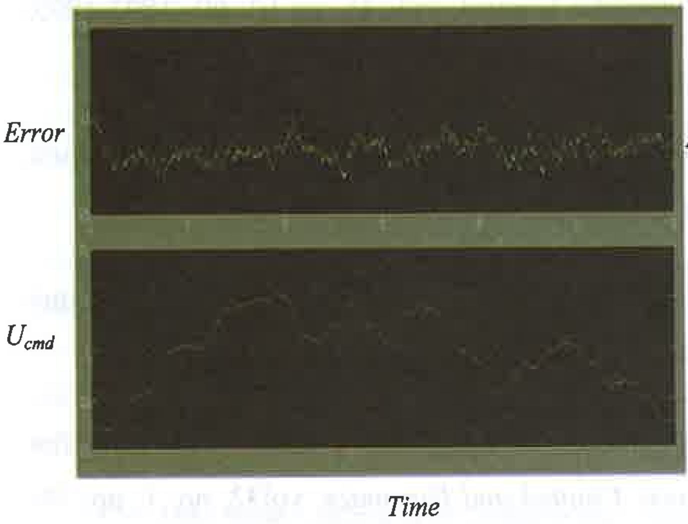


Figure 4. Behaviour of the System with Dynamic Inversion for Vbo=4Km/sec

Time to intersect depends on launch angle and velocity. Best results were delivered at 45° launch angle or when the weapon was pointed in the direction of the threat at launch. Once target is locked the χ_{LOS} will remain constant. The results demonstrate successful integration of adaptive control and trajectory optimization for the air defence system, also justifying intuitive judgment. The command input gradually tends to zero as the LOS is locked.

The control variable was the missed distance given by $M = R\sin(\chi_{LOS} - \chi)$ and the control law is given by $U = KPN.\dot{\chi}_{LOS}$. KPN represents the Proportional Navigation Constant. Usually $1 \leq KPN \leq 6$. Initial acceleration demand is high, but tends to zero towards the latter part of the flight as it tends to a constant bearing course, $\dot{\chi}_{LOS} = 0$. Even though large value of KPN leads to quicker interception, it also increases noise.

The adaptation gain, γ being a positive constant, is set to 2. The initial values of both parameters of the controller are chosen to be zero, indicating no priori knowledge. The initial conditions of both the plant and the model are set to zero as well. The simulation was again run for the adaptive control design. The results are as shown in figures 5 and 6, where it is seen that the controller responds well to external disturbances and drives the error to reach zero over a certain period of time.



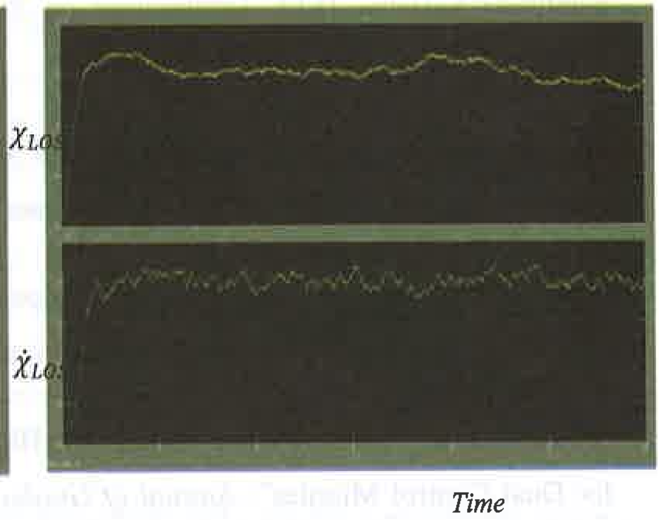


Figure 5. Controller Behaviour of the modified system
Performance and Parameter
Estimation of LOS angle for Vbo=4Km/sec

Figure 6. Tracking

Conclusion

The system is simulated in a two dimensional setting and the basic trajectory is optimized to meet the specified constraints. Control law is designed using dynamic inversion and adaptive control where good target tracking is delivered for improved performance of integrated interceptor control systems. The design enables the controller to adapt to perturbations in the threat trajectory thereby enabling quicker interception, i.e. at a location furthest from the home base.

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