Modification for End Area Rule Used in Volume Calculation of Highway Pavements

GI Wanasinghe[1], HKNA DE SILVA[1], PG Thennakoon[1], and KWGIM Kalugamuwa[1]

[1] *Kotelawala Defence University, Sri Lanka*
[#] Corresponding author; <wanasinghegi@yahoo.com>

Abstract— *Volume calculation of earth work of road pavements is very important section in project cost estimation. Practically it is very difficult to calculate the actual volume of a pavement as the cross section varies by its width, height, shape and formation level. The conventional calculation method is to use end area rule which calculates the area between two consecutive sections as the product of average area of sections and distance between sections assuming there is a linear variation of sectional area between two sections. The actual volume of the section deviates from the calculated volume using end area rule due to the violations of assumptions of the equation. This study was carried out to investigate on the errors of application of end area rule in volume calculations of highway pavements. The actual volume was calculated by a numerical analysis by integration and also by preparing a model using computer software. A modification factor was developed using the perpendicular distance between the centroids of cross sections and end area rule was applied with this modification factor. The validity of the results is evaluated by comparing the calculated volume with actual volumes obtained by mathematical and software models. It was concluded that a volume can be calculated up to a better accuracy using this modification factor together with the end area rule.*

Keywords— **Volume calculation, end area rule, modification factor**

I. INTRODUCTION

The road construction industry is very important arena a country looks forward as the transportation system is the key criterion of development. Volume calculation of highway pavement is the most difficult event of the cost estimation as it varies with the existing terrain. The volume is used not only to calculate the sub surface materials but also in post investigations such as material usage and workmanship.

The most widely used the most convenient method of calculating a volume is the use of cross sections measured at suitable intervals. This method leads the other techniques by ease of conduction as it does not require advance control surveying and can be performed with linear measurements and a level line. The volumes between cross sections are then calculated and the end area rule is the most widely used technique for this purpose. The end area rule is used to calculate the volume between two consecutive sections by multiplying the average area of the two cross sections and the perpendicular distance between the sections. This technique is valid only when there is a linear variation along the sections which is the basic assumption of the rule.

The above assumption is not considered in calculating the earth work volumes of road pavements as it is difficult to ensure the existence of linear variation. Furthermore, the perpendicular distance can not be calculated when the cross sections are with different shapes and heights. Therefore the volume calculated with end area rule would deviate from the actual volume creating errors in the result.

This study was performed to analyse the error of the volume due to non-parallelism of the measured distance between the sections and to introduce a modification factor in order to achieve a more accurate result with the conventional measurements. The study is limited to rectangular cross section with varying carriage width having the top surface in a horizontal plane. A numerical method was developed to calculate the actual volume of the section and it was confirmed with a digital model developed with a computer software. The results were then compared with the volume calculated with end area rule and a modification factor was introduced.

II. LITERATURE REVIEW

Estimation of area and volume is basic to most engineering schemes such as route, alignment, reservoirs, construction of tunnels. The excavation and hauling of material on such schemes is the most significant and costly aspect of the work on which the majority of project cost is depend on. There is no any method developed to calculate volumes perfectly but many researches to obtain a closure approximation.

Earthwork volumes have been conventionally estimated using the average end-area and the primordial method

(Hickerson 1964). Average end area rule has become the most widely used method among these as it requires simple linear measurements. But the main disadvantage of the method is the assumption of linear variation between the sections. Both above methods require cross-section areas to be of the same type as either cut or fill. Epps and Corey (1990) developed procedures to estimate earthwork volumes differently using end area rule for various configurations of cross section areas of cuts and fills. Cheng (2005) conducted a study to solve the inaccuracy problem caused by average end-area method and prismoidal method used for the calculation of roadway earthwork volume.

Cheng and Jiang (2013) reconfirmed the feasibility of average-end-area method for earthwork volume and the analysis of difference of accuracy between 3D method and average-end-area method. It shows that the critical value of interval distance between two consecutive cross sections is 30m for average-end-area method. Khalil (2015) stated that the average end area method is tedious and time consuming.

Many models for accurately estimating earthwork volumes have been intensively studied in literature. The average end area model and prismoidal model were commonly employed for estimating earthwork volumes. The prismoidal model gave an exact volume for linear profiles, while the average end area model generally overestimated the volume.

According to Cheng and Jiang (2019), reliable and accurate earthwork volume calculation is one of the most important components in roadway engineering that can influence the choosing of roadway alignment, the cost and construction. As the appearance and wide use of application of Digital Terrain Model (DTM), roadway design has stepped into 3D era and accordingly 3D method for earthwork volume calculation is also developed. But the concept of adopting average-end-area method which is considered as 2D method is deep-rooted in roadway design. It has been further investigated the accuracy comparison of Roadway Earthwork Computation between 3D and 2D Methods.

Goktepe and Lav (2003) developed a method called weighted ground elevation that considered the material properties in grade line selection to balance the cut and fill volume. All this researches were conducted by average-end-area method. Easa (1989 & 1992) introduced improvements in average-end-method when the studies on imprecision and limitations of 2D method in volume calculation are considered. Aruga and Akay (2005) developed a forest road design program based on a high-resolution Digital Elevation Model (DEM) from a light detection and ranging (LIDAR) system. After a designer had located the intersection points on a horizontal plane, the model firstly generated the horizontal alignment and the ground profile , and then it could precisely generate cross-sections and accurately calculate earthwork volumes using a high-resolution DEM. A shortage of this model was the incapability of properly optimizing horizontal and vertical alignments simultaneously. Li and Han (2007) used DTM to calculate cross section area, but still completed volume computation by 2D rule. These programs had begun to bring DTM into roadway design and volume calculation, but actually it can not be considered as pure 3D concept as they still use average-end-area or primodial method to compute earthwork volume finally.

III. METHODOLOGY

The scope of this study was limited to straight road segment having rectangular cross section with linearly varying road width. The volume of the section was calculated using numerical model and digital model. The results were compared with volume calculated by end area rule to introduce the modification factor.

Configuration of selected section is shown in Figure 01 and the linear dimensions used for the analysis are listed below.

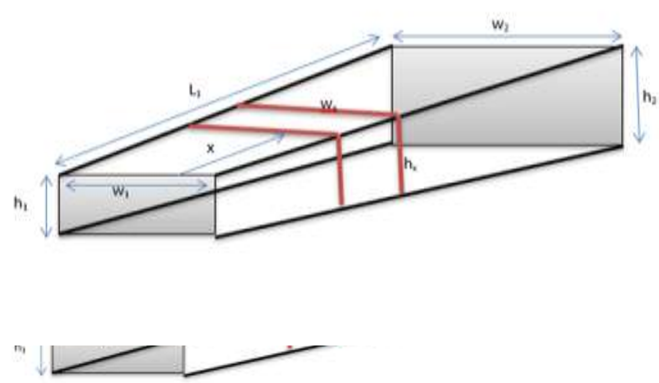

Figure 1. Section configuration

The symbols used are;

h_1, h_2 - heights of sections

w_1, w_2 - widths at each section

L - length between sections along Centre line

A. Numerical Model

An intermediate strip was selected as per the Figure 02 and its dimensions were calculated using linear interpolation. An equation for total volume is then developed by interpolation as mentioned below.

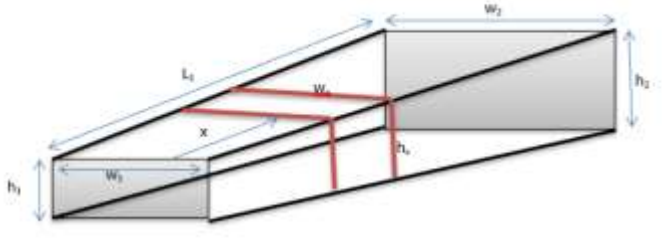

Figure 02. Dimensions of strip used for interpolation

$$w_x = w_1 + \frac{(w_2 - w_1)}{L} x$$

$$h_x = h_1 + \frac{(h_2 - h_1)}{L} x$$

$$\text{Volume} = \int_0^L h_x * w_x \, dx$$

$$= \int_0^L \left(h_1 + \frac{(h_2 - h_1)}{L} x\right) * \left(w_1 + \frac{(w_2 - w_1)}{L} x\right) dx$$

$$= \int_0^L \left[h_1 w_1 + \left[h_1 \frac{(w_2 - w_1)}{L} + w_1 \frac{(h_2 - h_1)}{L}\right] x + \left[\frac{(h_2 - h_1)}{L} * \frac{(w_2 - w_1)}{L}\right] x^2 \right] dx$$

$$= h_1 w_1 L + \frac{L}{2}[h_1 (w_2 - w_1) + w_1 (h_2 - h_1)] + \frac{L}{3}[(h_2 - h_1)(w_2 - w_1)]$$

$$= \frac{L}{6}[6h1w1 + 3(h1w2 - 2h_1w_1 + h_2w_1) + 2(h2w2 - h_2w_1 - h_1w_2 + h_1w_1)]$$

$$= \frac{L}{6}[2h_1w_1 + h_1w_2 + h_2w_1 + 2h_1w_2]$$

Digital Model

The pavement section illustrated in Figure 01 was generated using AutoCAD 2019 computer software which allows to measure the volume directly. Figure 03 shows the image of digital model with the volume given by the software.

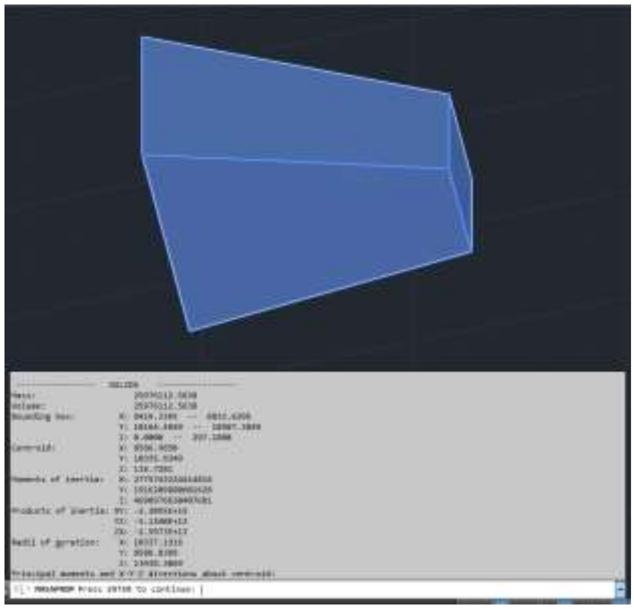

Figure 03. Digital model

The volumes calculated by both numerical and digital models were compared with volume given by end area rule for the error analysis. The ratio of volumes given by end area rule and numerical method were evaluated with the angle between two centroids of the cross sections in order to calculate the modification factor.

IV. OBSERVATION & RESULTS

A. Analysis of Volumes

10 samples with different section dimensions were used in the study and they are given in Table 1. .

Table 1. Section dimensions and volumes

Trial No.	**Dimension (inches)**				
	h_1	h_2	w_1	w_2	L
1	5.5	11.7	8.5	13.5	16.5
2	8.25	17.55	12.75	20.25	16.5

3	12.375	26.325	19.125	30.375	16.5
4	18.5625	39.4875	28.6875	45.5625	16.5
5	27.84375	59.23125	43.03125	68.34375	16.5
6	41.765625	88.846875	64.546875	102.515625	16.5
7	62.6484375	133.2703125	96.8203125	153.7734375	16.5
8	93.97265625	199.9054688	145.2304688	230.6601563	16.5
9	140.9589844	299.8582032	217.8457032	345.9902345	16.5
10	211.4384766	449.7873048	326.7685548	518.9853518	16.5

The volumes calculated by mathematical equation and the volume measured by digital method are given in Table 2 together with the volume calculated using end area rule.

Table 2. Volume measurements

Trial No	Volume (mm³)		
	Mathematical method	Digital method	End area rule
1	1603.525		1688.775
2	3607.93125		3826.040625
3	8117.845313		8549.423438
4	18265.15195		19326.20273
5	41096.59189		43281.45615
6	92467.33176		97383.27634
7	208051.4965		219112.3718
8	333726.2322		493002.8367
9	750884.0226		1109256.383
10	1689489.051		2495826.861

B. Error Analysis

It can be seen that the volumes given by mathematical mode and digital model are closer and hence can be considered as exact. But the value given by end-area-rule is deviating more from the actual value and the main reason for this deviation is suspected as due to the non-parallelism of the sections. Therefore the inclination of the line connecting the two centroids of the cross sections were calculated (Θ). The ratio of volumes between mathematical and end-area-rule was compared with inclination angle and the results are shown in Table 3 and Figure 4,

Table 3. Section dimensions and volumes

Trial No	Volume ratio	Ѳ	CosѲ
1	1.053164123	10’38’26.23’	0.982804689
2	1.060452753	15’44’19.69’	0.962508278
3	1.053164123	22’54’54.38’	0.921082754
4	1.058091539	32’22’44.51’	0.844523979
5	1.053164123	43’33’55.45’	0.7288179
6	1.048019653	54’58’21.98’	0.573965676
7	1.053164123	64’57’15.78’	0.423339747
8	1.47726726	72’41’50.06’	0.297420865
9	1.47726726	78’16’3.57’	0.203339975
10	1.47726726	83’28’16.34’	0.113702449

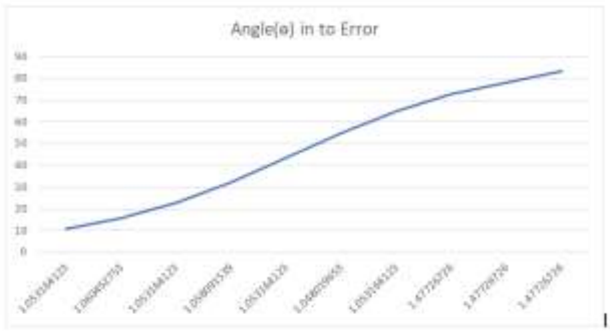

Figure 4. Variation of inclination Vs error ratio

V. CONCLUSION

By comparing the volumes given in Table 2, it can be concluded that the mathematical model developed is correct as it is confirmed by digital method.

By considering the linearity of the curve in figure 4, it can be stated that the x is linearly proportional and therefore it can be used as the modification factor for end area rule in order to achieve more accurate result with the conventional measurements.

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